*J. Fluid Mech.* (1974), *vol.* 64, *part* 3, *pp.* 611–622
*Printed in Great Britain*



# The jet density exponent issue for the noise of heated subsonic jets

By R. MANI

G. E. Research and Development Centre, P.O. Box 43, Schenectady, New York 12301

(Received 29 August 1973)

The subject of the present study is the question of how the sound power of a jet of constant exit velocity would vary if the jet exit density were varied. Changes in jet exit density would inevitably be accompanied in a real experiment by changes in the speed of sound (temperature) in the jet, so that both effects must be considered simultaneously. The point of view advanced at the end of the study is that experimentally observed results in this area seem to admit an explanation based on how the radiative efficiency of moving acoustic sources is affected by the shrouding effect of a jet flow whose velocity, temperature and density differ from those of the ambient fluid. This change in efficiency is calculated with the aid of a simple model as follows. We determine the acoustic power output of a convected monopole source, simple harmonic in its own frame of reference, moving along the axis of a plug-flow round jet whose velocity is the same as that of the source. The jet is doubly infinite and the source is assumed to have an infinite lifetime. The density and temperature of the jet are allowed to differ from those of the ambient fluid though the specific-heat ratio of the jet fluid is assumed to be the same as that of the ambient. The requirement of equality of the static pressure inside and outside the jet then calls for a certain restraint on how the jet density and temperature vary. For a specific value of the jet exit velocity, the variation of acoustic power with the ratio of jet to ambient density along with a simple assumption on how the source strength varies with jet density are employed to deduce theoretically the 'jet density exponent' for jets which are subsonic with respect to the ambient speed of sound. The jet density exponent is found to depend both on the jet Mach number and even more strongly on a source frequency parameter. The theoretical results are compared with some experimental studies of this problem. Encouraging agreement is obtained both for the detailed observed effects on the power spectrum and the exponent for the overall power.

## 1. Introduction

The most useful results in the area of jet noise at the present time still stem from the point of view advanced by Lighthill (1963), who ascribed jet noise to convected quadrupoles derived from solenoidal velocity fluctuations associated with the turbulence in the jet flow. A crucial step employed by Lighthill in extending his theory to high subsonic Mach numbers (the term 'Mach number' in this study will always refer to the ratio of the jet velocity to the ambient speed of sound) was the use of convected rather than stationary sources. The reader must refer to the original papers by Lighthill for a full exposition of the detailed reasoning behind this step but this major advance enables one to (*a*) retain source compactness at the higher subsonic Mach numbers, (*b*) avoid the pitfall of artificially inflating the rate of change of the turbulence (a frozen convected pattern of turbulence radiates no sound) and (*c*) realize that, at subsonic Mach numbers, it is the decay of turbulence in its own frame of reference that generates jet noise. In his work, however, Lighthill implicitly assumes that the jet flow itself is also acoustically compact, so that, in the detailed calculations, the convected quadrupoles are assumed by him to radiate freely into an ambient atmosphere. Such a treatment suppresses the refraction of the sound by the jet flow and also any effect that the shrouding of the sources by the jet flow may have on their radiative efficiency.

These deficiencies of the freely convected quadrupole fomulation of Lighthill have, of course, received attention in several papers, e.g. Ribner (1960, 1962), Powell (1960), Csanady (1966) and most notably Phillips (1960). Phillips, in particular, took the crucial step of identifying the need to abandon the analogy with stationary-media acoustics of Lighthill and to study the high-speed jet-noise problem using a convected wave equation for a medium with variable mean velocity, temperature, etc. Phillips' approach succeeds not only in automatically accounting for the jet flow shrouding the sources but also in providing a logical basis for approximating the quadrupole source-strength term $\rho u_i u_j$ in the equation for aerodynamic noise by $\rho_J u_i u_j$, where $\rho_J$ is the density of the jet fluid. As noted by Phillips, this approximation is untenable at high speeds for the source term in Lighthill's equation. In a recent study, Doak (1972) has pointed out that even the Phillips equation contains a certain confusion between source-like and propagation terms. He has pointed out that a higher-order (third-order) equation identical to the one used to study wave propagation in parallel shear flows in a duct is needed to study jet-noise problems. However it is not necessary to resort to such a higher-order equation in the case of the plug-flow jet.

The difficulty with replacing the analogy approach by one based on a convected wave equation is the loss of all the powerful apparatus of classical acoustics. This difficulty is illustrated in Phillips (1960), where only a high frequency analysis is attempted, making the analysis valid in principle for high supersonic jet velocities. A recent careful experimental study of subsonic jet noise by Lush (1971) has built up a case for incorporation of a jet flow shrouding the sources even in the subsonic case. Motivated mainly by this study, Mani (1972) studied the shrouding effect of a plug-flow jet whose density and temperature were the same as those of the ambient fluid on the power output of a moving monopole source. Over the whole frequency range of interest for jet noise, very significant differences in the power output from what might be computed on the basis of a freely moving source model (i.e. a source moving through a stationary medium) were found at jet Mach numbers greater than 0·7. In the case of a freely moving mass source of strength $q_0$ ($q_0$ would have the dimensions mass per unit time) oscillating in its frame of reference at frequency $\omega_0$, with $\rho_0$ and $c_0$ denoting the density and speed of sound of the ambient fluid, the power output when the source is convected at a Mach number $M$ may be shown to be $q_0^2\omega_0^2/8\pi\rho_0 c_0(1-M^2)^2$. The factor $(1-M^2)^{-2}$ represents an augmentation of the power output of the moving source as compared with the output of a stationary source. This effect is often referred to as convective amplification in jet-noise theory. Lighthill's approach of neglecting the shrouding effect of jet flow by dealing with freely convected sources leads to a convective amplification factor which is frequency independent whereas calculations of the type reported in Mani (1972) incorporating the shrouding effect of the jet flow lead to a convective amplification factor which is strongly frequency dependent. At high frequencies, conjectures of Ribner (1960), Powell (1960) and Csanady (1966) are borne out, namely, that the radiated sound power output of the source exhibits no convective amplification at all. Physically, we may explain this by noting that at high frequencies we expect the source to 'sense' only the nearby ambient fluid relative to which it is not moving, and hence there would be no convective amplification. In general, the convective amplification decreases with increasing frequency. The study reported in Mani (1972) is both in accord with the results of Lush (1971) and also explains another result of jet-noise experiments, which is the failure of the peak frequency of the jet-noise power spectrum to scale linearly with velocity (as might be expected from notions of Strouhal scaling).

A more notable difficulty with the Lighthill analysis appears when one deals with cases where the jet density and temperature differ from those of the ambient fluid. Such a situation arises commonly in the case of a heated jet. If a freely moving source model of the Lighthill type is employed, the jet density appears only in terms of the strength of the quadrupole, which is $\rho u_i u_j$. Thus the most simple-minded expectation based on the Lighthill theory would be that the jet power varies as $\rho_J^2$ (at constant jet velocity). In Lighthill (1963), it is pointed out that the quadrupole strength is probably proportional to $\frac{1}{2}(\rho_J+\rho_0)$, where $\rho_0$ is the ambient density. On the basis of this, Lighthill suggests a dependence of sound power on $\rho_J$ somewhere between $\rho_J^2$ and $\rho_J$. Results of a very careful series of experiments on this subject have recently been published by Hoch *et al.* (1972). The exponent $n$ in the relation between the power $P$ and the jet density (i.e. $P \propto \rho_J^n$ at constant jet velocity) is popularly referred to in the literature as the jet density exponent. Hoch *et al.* have determined experimentally that $n$ is a function of the jet Mach number. The Lighthill theory, with its neglect of the shrouding effect of the jet flow and its replacement of the source-strength term $\rho u_i u_j$ by $\rho_J u_i u_j$, could probably be regarded as a valid low Mach number theory. Paradoxically, the work of Hoch *et al.* shows that it is in the supersonic Mach number range of about 1·3–2·5 that $n$ approaches 2. For jet Mach numbers in the range 0·5–1·3 the index is less than 2, sometimes even negative. Moreover, their power spectral results show that heating (changing $\rho_J$ and $c_J$) affects different portions of the power spectrum differently. In general, heating raises the low frequency portions and depresses the high frequency portions of the power spectrum.

Motivated principally by this study of Hoch *et al.*, the calculations reported in Mani (1972) are extended in the present study to the case where the jet density and temperature differ from those of the ambient fluid. The intent is to see how far one can get in explaining the available data by resorting to purely acoustic explanations, i.e. to explore the effect of a mismatch of mean velocity, temperature and density between the fluid inside a jet and that outside it on the radiative efficiency of a source moving inside the jet. Obviously heating will introduce other changes in a jet flow such as altering the mixing characteristics, introducing temperature or entropy spottiness, etc. The success of the present study in explaining the data of Hoch *et al.* does suggest, however, that the primary explanation of the difference between the noise of a cold as opposed to a heated jet may be the acoustic effect.

In concluding this introduction, we note that the density exponent issue, while primarily of interest from a theoretical standpoint, has also some modest practical significance. Since the thrust of a jet varies as $\rho_J$, if the index for the total power were greater than 1, a simple means of lowering jet-noise power at constant exit velocity, nozzle area and thrust would be to heat the jet. Conversely, if $n < 1$, heating would increase the radiated acoustic energy at constant thrust. Another reason for practical interest in the subject is, of course, the need to scale the noise from hot and cold jets.

## 2. Formulation and solution of model problem

The model problem is sketched in figure 1. We wish to determine the sound field due to a fluctuating monopole point source translating at a uniform subsonic velocity $U$ (where $U < c_0$, the speed of sound in the ambient fluid). The source translates along the axis of a round jet whose velocity profile we assume to be a plug flow. Also the jet velocity is taken to be equal to that of the source.

The source is assumed to have a time dependence $q_0 \cos(\omega_0 t)$ in its own frame of reference. The mean jet density and speed of sound are $\rho_1$ and $c_1$ while those of the ambient fluid are denoted by $\rho_0$ and $c_0$. Now the static pressure inside the jet is given by $p = \rho_1 c_1^2/\gamma_1$ and similarly, that of the ambient fluid (air) by $\rho_0 c_0^2/\gamma_0$. Since the static pressures inside and outside the jet must be equal, if we assume that $\gamma_1 = \gamma_0$ (a reasonable assumption for the heated-jet situation but less valid if a gas other than air, such as freon, etc., is used for the jet fluid), then we must have $\rho_1 c_1^2 = \rho_0 c_0^2$ to balance the static pressures. This implies a coupling between the density ratio and speed-of-sound ratio (i.e. $\rho_1/\rho_0 = (c_1/c_0)^{-2}$), which is always employed in the current study.

We wish to determine analytically an acoustic velocity potential $\phi$ which satisfies in region I of figure 1 (outside the jet)

$$\nabla^2\phi - c_0^{-2}\phi_{tt} = 0, \ldots, \tag{1}$$

and in region II (inside the jet) of figure 1,

$$(1-M_1^2)\,\phi_{xx} + \nabla_2^2\phi - 2\frac{M_1}{c_1}\phi_{xt} - \frac{\phi_{tt}}{c_1^2} = \frac{q_0}{\rho_0}\cos(\omega_0 t)\,\delta(x-Ut)\,\delta(y)\,\delta(z), \ldots, \tag{2}$$

where $M_1 = U/c_1$ and $\nabla_2^2$ stands for the Laplacian operator in the $y$, $z$ plane. We assume that $\rho_1 \leqslant \rho_0$ (in view of the interest in heated jets), so that $c_1 \geqslant c_0$, and

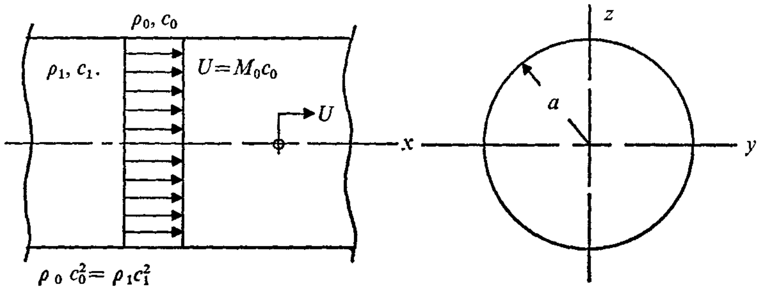


FIGURE 1. Model problem for the noise of heated jets.

hence if $U < c_0$ then $U$ is also less than $c_1$. This means that $M_1 < 1$. At the jet–still-air interface (i.e. at $r = a$), we require the following.

(*a*) Continuity of acoustic pressure $p$, that is

$$p = \begin{cases} -\rho_0 \phi_t \text{ in region I,} & (3) \\ -\rho_1\{\phi_t + U\phi_x\} \text{ in region II.} & (4) \end{cases}$$

(*b*) Continuity of the radial acoustic particle displacement, say $\eta$, so that

$$\phi_r = \begin{cases} \eta_t \text{ in region I,} & (5) \\ (\eta_t + U\eta_x) \text{ in region II.} & (6) \end{cases}$$

Outside the jet, i.e. in region II, the velocity potential $\phi$ is also subject to a radiation condition which states that only outgoing waves are emitted by the moving source.

Before proceeding further, it is necessary to discuss the implications of employing a plug-flow or top-hat velocity profile, which is known to be unstable when excited by certain wavenumber–frequency combinations of longitudinally travelling waves. The general procedure for examining the stability (e.g. Batchelor & Gill (1962) consider the incompressible case) is to consider the unforced jet eigenvalue problem. In other words, a solution of the form

$$R(r) \exp [i(kx - \omega t)]$$

is assumed for $\phi$, whence, in the absence of any source term driving the system, specifying real $\omega$ determines $k$ as a function of $\omega$ (spatial stability analysis) or alternatively specifying real $k$ determines $\omega$ as a function of $k$ (temporal stability analysis). The imaginary parts of $k$ or $\omega$ respectively determine the regimes of instability. In the present problem a source of the form $\cos(\omega_0 t)\,\delta(x - Ut)$ imposes a specific type of travelling-wave disturbance on the jet column. On Fourier decomposition of the source excitation, the travelling waves turn out to be of the form $\exp i[(\omega \mp \omega_0) x/U - \omega t]$ with real $\omega$. Thus formal consideration of the problem of a source of infinite lifetime yields simple travelling-wave excitation of the jet column which produces either a propagating or a decaying sound field outside the jet depending on whether the wave speed $|U[1 \mp \omega_0/\omega]^{-1}|$ parallel to the jet axis exceeds $c_0$ or not. This then leads to the obvious result that effective acoustic power is produced in the far field over a frequency range $\omega_0/(1 + M_0) \leqslant \omega_0 \leqslant \omega_0/(1 - M_0)$, where $M_0 = U/c_0$ (Morse & Ingard 1968) as predicted by the Doppler-shift formula.

It seems likely, however, that if one sought the solution to the problem as the limit of an initial-value problem, i.e. assumed a source strength of the form $\delta(y)\,\delta(z)\cos(\omega_0 t)\,\delta(x-Ut)\,H(t-t_0)$, where $H(t)$ is the unit step function, and then studied the limit of the solution as $t_0 \rightarrow -\infty$, one would find that the 'starting up' process of the source triggers Kelvin–Helmholtz instability at the jet–still-air interface. (If the source were assumed to be switched on suddenly at $t = t_0$, it would be necessary for $\phi \equiv 0$ (or constant) for $t \leqslant t_0$. Ensuring this requires that in the Fourier integral representation of $\phi$ the path of integration in the $\omega$ plane be specified in a certain manner. Presumably, then, in deforming that path of integration onto the real-$\omega$ axis ($-\infty < \omega < \infty$), unstable pole contributions corresponding to the excitation of the instability modes would be picked up in addition to the contribution from integration over the real-$\omega$ axis, which alone is discussed herein.)

This aspect of the problem, while undoubtedly a difficulty with the plug-flow profile or indeed any mean velocity profile that is inflexional, is ignored in the present study on the basis of the following physical argument. In practice, jets (at sufficiently high Reynolds numbers) do represent a stable flow situation though characterized by high turbulence levels (the r.m.s. turbulence level can often be as high as 15 % of the jet velocity). Real jet flow then is a flow with a distribution of mean velocity and turbulence levels which is manifestly stable to source excitations of the type that lead to jet noise. The high turbulence level in the jet itself could be a stabilizing agent by a mechanism involving eddy viscosity as has been noted in several previous studies of turbulent shear flows. For example, Landahl (1967) surmized that even for turbulent boundary layers (generally characterized by lower turbulence levels than a jet flow) the eddy viscosity seemed to be about 80 times as effective as the molecular viscosity. Similarly Bishop, Ffowcs Williams & Smith (1971) specifically suggested, with respect to high-speed jets, the substantial lowering of the effective jet Reynolds number due to the eddy viscosity. The justification for the use of a plug-flow velocity profile, then, rests on the fact that, in attempting to infer the effect of the more complicated mean velocity profile of the true jet on the radiative efficiency of a moving source, one may exploit the relatively low frequency nature of jet-noise sources to argue that the precise nature of the real velocity profile need not be retained. Since we know that the real jet flow is stable and it is understood that the plug-flow profile is only employed as an analytical artifice to assess conveniently the shrouding effect of the flow on the radiative efficiency of the source, we may then reject the unstable excitation of the plug-flow profile owing to the starting-up process of the source (which appears in the rigorous analytical solution of (1) and (2) when a proper initial-value problem is posed) as not germane to the real physical problem. In what follows, then, we shall deal only with the stable and bounded solution to (1) and (2) subject to the matching conditions (3)–(6) assuming a source of infinite lifetime. The problem posed by (1) and (2) is a transient one and the required bounded solution may be obtained by formally applying the Fourier integral method taking $\omega$ real. A similar difficulty arises, of course, in the calculations of Mani (1972) though the arguments for ignoring the stability issue were not spelled out in as much detail as above. From a fluid-mechanical rather than a mathematical point of view the case for both the present calculations as well as those of the earlier study (Mani 1972) must perhaps be judged on the degree to which the results are in accord with physically observed features of jet noise.

The analysis is very similar to that given in Mani (1972) and in the interests of brevity only the final result is stated. The acoustic power spectrum extends over the Doppler-shifted frequency range $\omega_0/(1+M_0) < \omega < \omega_0/(1-M_0)$ and is given by

$$16\pi\rho_0 U\omega|A_I^+|^2 = I(\omega), \tag{7}$$

where $A_I$ is given by

$$A_I^+ = \frac{q_0\kappa^+[Y_0(\kappa^+a)J_1(\kappa^+a) - Y_1(\kappa^+a)J_0(\kappa^+a)]}{16\pi\rho_0\left[\dfrac{\omega}{\omega_0}\kappa^+H_0^{(1)}(k^+a)J_1(\kappa^+a) - k^+\dfrac{\rho_1}{\rho_0}\left(\dfrac{\omega_0}{\omega}\right)J_0(\kappa^+a)H_1^{(1)}(k^+a)\right]U} \tag{8a}$$

if $\omega \in [\omega_0(1-M_1), \omega_0(1+M_1)]$, and

$$A_I^+ = \frac{-q_0\bar{\kappa}^+[K_0(\bar{\kappa}^+a)I_1(\bar{\kappa}^+a) + I_0(\bar{\kappa}^+a)K_1(\bar{\kappa}^+a)]}{8\pi^2\rho_0 U\left[\dfrac{\omega}{\omega_0}\bar{\kappa}^+I_1(\bar{\kappa}^+a)H_0^{(1)}(k^+a) + \dfrac{\rho_1}{\rho_0}\dfrac{\omega_0}{\omega}k^+I_0(\bar{\kappa}^+a)H_1^{(1)}(k^+a)\right]} \tag{8b}$$

otherwise. The total power is given by

$$P = \int_{\omega_0/(1+M_0)}^{\omega_0/(1-M_0)} I(\omega)\,d\omega. \tag{9}$$

Note that, in (8), $k_0 = \omega/c_0$, $k_1 = \omega/c_1$ and

$$k^{+2} = \left(\frac{1-M_0^2}{M_0^2}\right)\left[\left(\frac{\omega_0/c_0}{1-M_0} - k_0\right)\left(k_0 - \frac{\omega_0/c_0}{1+M_0}\right)\right], \tag{10}$$

$$\kappa^{+2} = \frac{1}{M_1^2}\left\{\left[\frac{\omega_0}{c_1}(1+M_1) - k_1\right]\left[k_1 - \frac{\omega_0}{c_1}(1-M_1)\right]\right\}, \tag{11}$$

$$\bar{\kappa}^{+2} = -\kappa^{+2} = \frac{1}{M_1^2}\left\{\left[k_1 - \frac{\omega_0}{c_1}(1+M_1)\right]\left[k_1 - \frac{\omega_0}{c_1}(1-M_1)\right]\right\}. \tag{12}$$

## 3. Computed results and inferences

The power calculated in (20) is non-dimensionalized first by the power of a freely moving source, which is $q_0^2\omega_0^2/8\pi\rho_0 c_0(1-M_0^2)^2$. With $M_0$ and $\omega_0 a/\pi U$ fixed, the non-dimensional power (say $P'$) is computed for $\rho_1/\rho_0$ ranging from 0·3 to 1·0. By constructing a curve of $\log P'$ against $\log(\rho_1/\rho_0)$ by the method of least squares an exponent $n'$ is determined for each $M_0$ and $\omega_0 a/\pi U$. $\omega_0 a/\pi U$ may be termed a source Strouhal number (e.g. as in Lush 1971). Now the source strength $q_0$ itself will vary linearly with the jet density whether one uses the quadrupole model of Lighthill (1963) or the fluid dilatation model of Ribner (1962). Since $P'$ itself varies as $\rho_1^{n'}$ and $P'$ is the power normalized by $q_0^2$, with $q_0 \sim \rho_1$, one would then expect the actual power to vary with the density $\rho_1$ to the power $n'+2 = n$.

This theoretically deduced exponent $n$ is plotted in figures 2(*a*)–(*c*) for $M_0 = 0{\cdot}5$, 0·7 and 0·9 and source Strouhal numbers in the range 0·1–1·0. For source Strouhal numbers greater than 1, owing to the high frequencies involved (high $\omega_0 a/c$), the plug-flow model would be less adequate.

If we take the limit of (8*a*) or (8*b*) at very low frequencies we can readily show

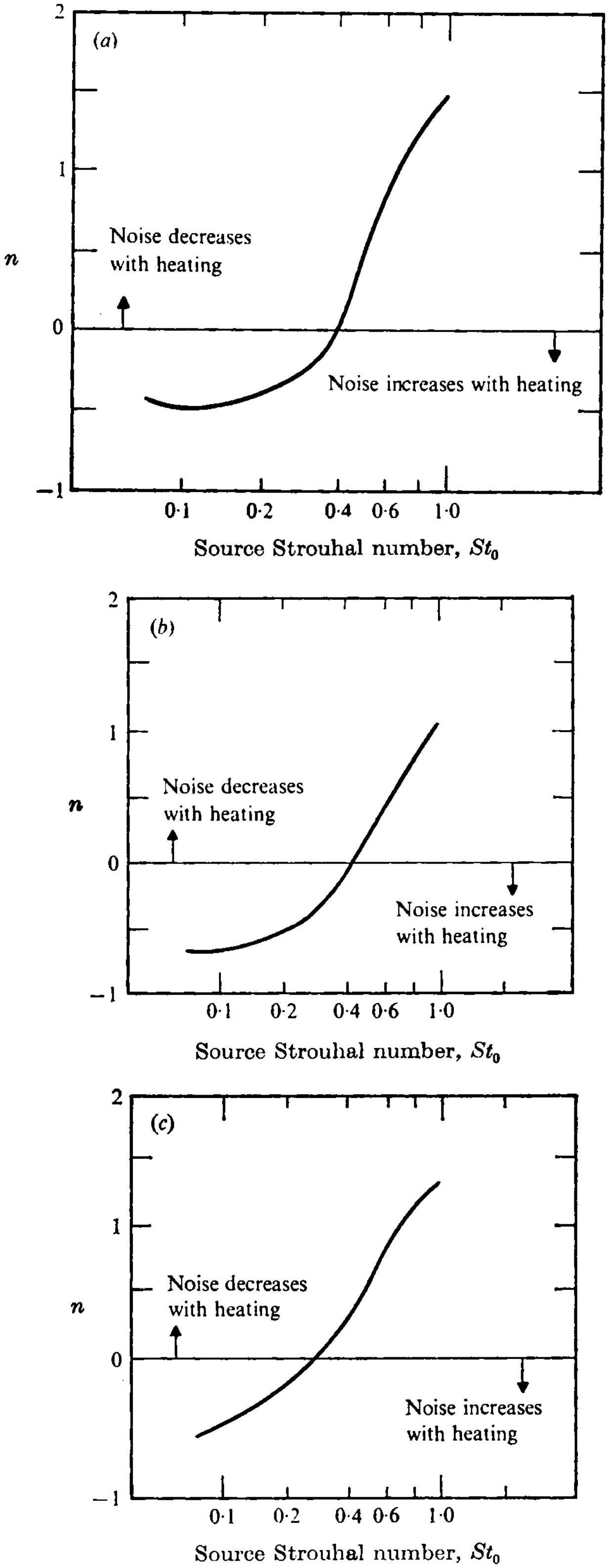


FIGURE 2. $n$ as a function of $St_0$ for (a) $M_0 = 0{\cdot}5$, (b) $M_0 = 0{\cdot}7$ and (c) $M_0 = 0{\cdot}9$.

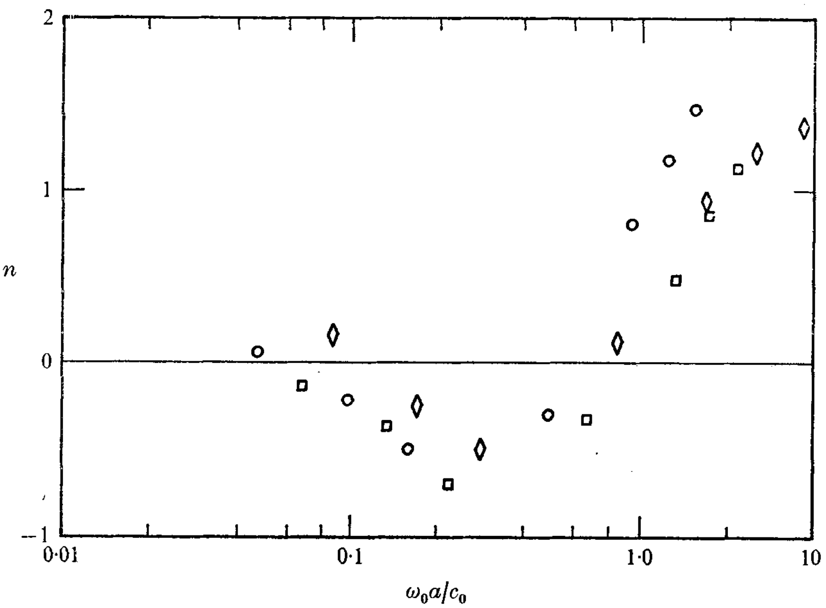


FIGURE 3. $n$ as a function of $\omega_0 a/c_0$ for various $M_0$. ○, $M_0 = 0{\cdot}5$; □, $M_0 = 0{\cdot}7$; ◇, $M_0 = 0{\cdot}9$.

that the index $n$ would be expected on the basis of this model to tend to zero. In general, then, for subsonic Mach numbers, the present model predicts that $n \rightarrow 0$ as the frequency parameter approaches zero, is then negative for a range of frequencies and finally starts increasing monotonically with frequency. It is not possible to extract analytically a high frequency limit from (8*a*) and (8*b*) but if one used the argument that at high frequencies the source output is determined only by its own immediate surroundings, the exponent would depend on how $q_0^2/\rho_1 c_1$ varies with $\rho_1$, and since $q_0 \sim \rho_1$ and $c_1 \sim \rho_1^{-\frac{1}{2}}$, for a monopole source model, the exponent would tend to 1·5 at high frequencies. The intrinsic source distributions generating jet noise do exhibit Strouhal scaling with respect to velocity (this is confirmed either by measurements inside the jet or by looking at the 90° point far-field data, where convective–refractive effects are absent), so that high speeds do correspond to high real-source frequencies and vice versa. In figure 3, the exponents for $M_0 = 0{\cdot}5$, $0{\cdot}7$ and $0{\cdot}9$ are shown as a function of a real-frequency parameter $\omega_0 a/c_0$. There is a general trend towards exponents of value zero as $\omega_0 a/c_0 \rightarrow 0$, followed by a region of negative exponents and a tendency for $n$ to attain values of 1·5 for high values of $\omega_0 a/c_0$ almost independent of the jet Mach number. In view of the Strouhal scaling with respect to velocity exhibited by the source distributions, one would expect (in terms of figure 3) the higher jet velocities to correspond to higher values of $\omega_0 a/c_0$. ($\omega_0 a/c_0$ is $\pi M_0$ times the source Strouhal number.) In other words figure 3 indicates that even in terms of jet velocities one would expect a changing exponent (say for the total power) starting off at zero at the lowest velocities, then becoming negative and then finally increasing monotonically with velocity. The present calculations are of course limited to subsonic jet velocities.

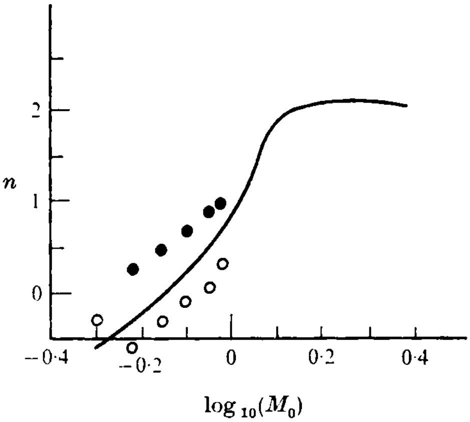


FIGURE 4. Comparison with data from Hoch *et al.* (1972). Current study: ○, $St_0 = 0{\cdot}3$; ●, $St_0 = 0{\cdot}6$. ——, experimental curve from figure 17 of Hoch *et al.*

In figure 4, we show first the empirical result obtained for the exponent $n$ for the total power obtained by Hoch *et al.* (1972). As indicated earlier, Hoch *et al.* found that $n$ is a function of the jet Mach number. In order to compare the present analysis with the data of Hoch *et al.* it is necessary to estimate source Strouhal numbers representative of the total power. On the basis of jet noise at low Mach numbers (where refractive, shrouding and Doppler-shift effects should be negligible), it was felt that a source Strouhal number somewhere between 0·3 and 0·6 would represent a 'typical' source Strouhal number for assessing an exponent for the total power. Shown in figure 4, for $M_0$ in the range 0·5–0·95, are results for $n$ of the present study for source Strouhal numbers of both 0·3 and 0·6. Except at the lowest Mach number 0·5, the predicted values of the exponent for source Strouhal numbers of 0·3 and 0·6 bracket the experimental values of Hoch *et al.* quite well.

As noted earlier, at the lower Mach numbers (and associated low real frequencies), the present analysis predicts that the exponent tends to zero. This result is apparently at variance with the SNECMA–NGTE study of Hoch *et al.* There are two points to note in this regard.

(i) The present study needs to be extended to higher-order multipoles as well as to sources convecting at velocities different from (less than) the jet speed. Extension to finite source lifetimes is also needed. It is not clear how much these extensions will alter the theoretical predictions of the exponent.

(ii) It is in the lower velocity range that isolation of the jet density exponent associated with pure jet noise becomes most difficult from an experimental point of view. This is because of the ever-present danger of internal noise sources (termed 'parasitic' noise by Hoch *et al.*) such as valve noise, combustion noise, etc. One can easily show that the effect of such a combustion noise source will be to lower the effective index from its value for pure jet noise in an experimental situation. In Hoch *et al.* (1972) it is pointed out that the NGTE group worked at the low velocity end while the SNECMA group worked at the high velocity end. Hoch *et al.* have cited the good agreement between the results of the two groups in the region of overlap of velocities as one indication of the internal cleanliness of

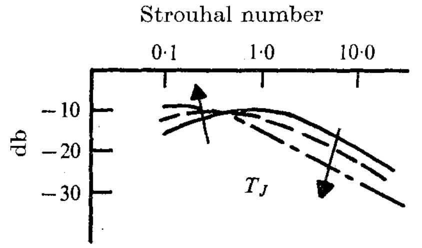


FIGURE 5. Effect of heating on power spectrum: figure 19 of Hoch *et al.* (1972), $M_0 = 0{\cdot}6$. —·—, $P_J/P_0 = 1{\cdot}08$, $T_J = 913$ °K; ---, $P_J/P_0 = 1{\cdot}15$, $T_J = 482$ °K; ——, $P_J/P_0 = 1{\cdot}24$; $T_J = 344$ °K.

their facility. However they do point out that the region of overlap extends from $M_0 = 0{\cdot}6$ upwards, so that "pure jet noise is being measured, at least, above jet velocities of 200 m/s" ($M_0 = 0{\cdot}6$). We may note that an earlier experimental study by Rollin (1958) did conclude that the density exponent was zero.

A reviewer contends that other unpublished experimental evidence indicates that the jet density exponent continues to become more and more negative as jet velocities decrease. He has referred to an explanation to be published by Morfey (1973) concerning the 'radiation by turbulence in the presence of density inhomogeneities'. It should be said that the present study does account for the density difference between $\rho_1$ and $\rho_0$, which appears, for example, in the dynamical condition (13). In other words, the effect on the radiation by the turbulence of both the mean temperature (speed of sound) and the mean density mismatch is simultaneously allowed for. If however Morfey's argument pertains to a new source of sound radiation whose strength *increases* with decreasing density and increasing temperature, that would indeed explain the observed jet density exponent at the low jet velocities. The present paper's calculations should be valid for a source whose strength is proportional to the jet density.

Finally, in figure 5, we show the detailed effects on the power spectrum due to heating observed by Hoch *et al.* at a velocity corresponding to $M_0 = 0{\cdot}6$. It is observed that they find that heating increases the low frequency portions of the spectrum while depressing the higher frequencies. The SNECMA–NGTE power spectra are actually relative power spectra (i.e. differences between third octave and overall powers). However figure 4 shows that at $M_0 = 0{\cdot}6$ the observed index for total power was very close to zero, thus implying that the overall power changed very little as the jet density was varied. This means that the results in figure 5 are also essentially representative of absolute changes in power spectra, which is what is predicted in figure 2. This is fully in accord with figure 2, wherein (as indicated) indices greater than zero correspond to portions of the frequency spectrum lowered upon heating and indices less than zero to portions raised by heating.

## 4. Conclusions

It appears from the present study that the differences in noise between a heated and a cold jet do have largely an acoustic explanation, being attributable to the effect on the radiative efficiency of a moving source due to the mismatch of the velocities, densities and temperatures inside and outside the jet. The fact that figure 5, taken from Hoch *et al.* is in accord with figure 2 appears to be the most impressive evidence of this for it is difficult to conceive of other explanations based on entropy fluctuations, jet mixing, etc., that would explain the tendency of heating to raise the low frequency end of the power spectrum while depressing the high frequency end.

As noted earlier, extensions to higher-order multipoles and to sources of finite lifetime are undoubtedly needed but the least the present study may be said to achieve is to indicate the profit of pursuing such analyses. The main object of the study is really to pinpoint the need to incorporate mean flow shrouding effects on the radiation by the sources, a matter which necessitates the formulation of jet-noise problems in the framework of convected wave equations rather than the Lighthill analogy form. The results in figures 2–5 are certainly encouraging as they indicate possible direct relevance of the present calculations to jet noise but such a conclusion can only be tentative at this stage. The chief result is the inadmissibility of regarding the jet flow as acoustically compact despite the 'low frequency' nature of jet noise.

I am deeply indebted to I. H. Edelfelt for help in programming the calculations reported herein. I also wish to acknowledge some very helpful correspondence and conversations with Professor J. E. Ffowcs Williams on the subject of this paper, particularly concerning the implications of the unstable solutions to (1)–(6). Financial support of this study by the U.S. Air Force and the U.S. Department of Transportation is gratefully acknowledged. A preliminary version of this study was presented at the AGARD specialists' meeting on 'Noise Mechanisms' held at Brussels, Belgium, 19–21 September 1973.

## REFERENCES

BATCHELOR, G. K. & GILL, A. E. 1962 *J. Fluid Mech.* **14**, 529.
BISHOP, K. A., FFOWCS WILLIAMS, J. E. & SMITH, W. 1971 *J. Fluid Mech.* **50**, 21.
CSANADY, G. T. 1966 *J. Fluid Mech.* **26**, 183.
DOAK, P. E. 1972 *J. Sound Vib.* **25**, 263.
HOCH, R. G., DUPONCHEL J. P., COCKING, B. J. & BRYCE, W. D. 1972 *J. Sound & Vib.* **28**, 649.
LANDAHL, M. J. 1967 *J. Fluid Mech.* **29**, 441.
LIGHTHILL, M. J. 1963 *A.I.A.A. J.* **1**, 1507.
LUSH, P. A. 1971 *J. Fluid Mech.* **46**, 477.
MANI, R. 1972 *J. Sound Vib.* **25**, 337.
MORFEY, C. L. 1973 *J. Sound & Vib.* **31**.
MORSE, P. M. & INGARD, K. U. 1968 *Theoretical Acoustics*. McGraw-Hill.
PHILLIPS, O. M. 1960 *J. Fluid Mech.* **9**, 1.
POWELL, A. 1960 *J. Acoust. Soc. Am.* **32**, 1609.
RIBNER, H. S. 1960 *J. Acoust. Soc. Am.* **32**, 1159.
RIBNER, H. S. 1962 *UTIAS Rep.* no. 86.
ROLLIN, V. G. 1958 *N.A.C.A. Tech. Note*, no. 4217.